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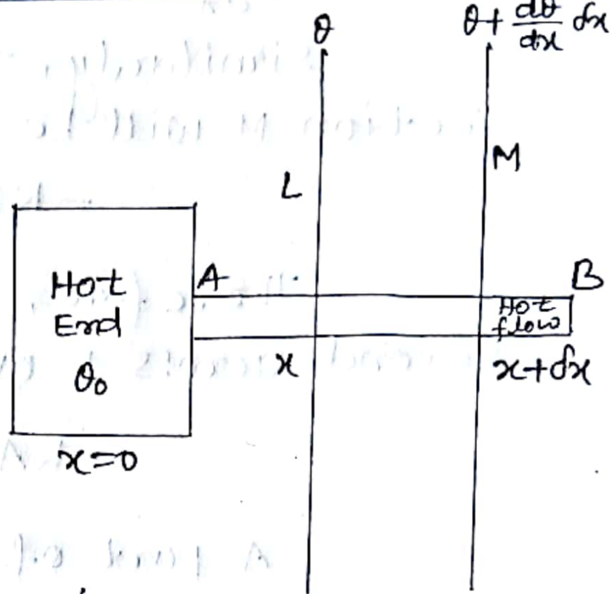
B.Sc (Part-I) Hons
paper-II

Fourier Equation of heat flow.



Consider a long uniform metal bar, whose one end is maintained at a constant temperature θ_0 relative to the temperature of the surroundings. Let θ is the temperature at a distance x from the hot end and $\frac{d\theta}{dx}$ is the temperature gradient.

In the begining the temperature at various points along the bar gradually rise. In steady state, the temperatures become stationary. A steady flow of heat now takes place the bar. If the bar is exposed to the surrounding, a part of the heat flowing across any cross-section of the bar escapes out from the sides. Hence, smaller and smaller amounts of heat pass across successive sections.

Now consider two cross section of the bar, L and M, at the distances x and $x+dx$ from the hot end. Now, before the steady state is reached the temperature at the section of the bar is increasing. Let θ is the excess of the temperature at the section of L at any instant t. Then, if $\frac{d\theta}{dx}$ is the temperature gradient at L, the excess temprature at M will be $\theta + \frac{d\theta}{dx}\delta x$

If A is the area of cross-section of the bar and K is the thermal conductivity of its material then the rate of heat flow across the section L will be $-KA\frac{d\theta}{dx}$.

Similarly, the rate of heat flow across the section M will be

$$-KA\frac{d}{dx}\left(\theta + \frac{d\theta}{dx}\delta x\right)$$

Therefore, the excess of heat flowing per second across L over that flowing across M is

$$-KA\frac{d\theta}{dx} - \left[-KA\frac{d}{dx}\left(\theta + \frac{d\theta}{dx}\delta x\right)\right] = KA\frac{d^2\theta}{dx^2}\delta x \quad \text{---(1)}$$

A part of this heat is absorbed by the element of the bar between L and M, and lost from the sides of the element to the surroundings. If ρ is the density, c is the specific heat of the material of the bar and $d\theta/dt$ is the rate of rise in temperature, then the heat absorbed per second by the element will be given by

$$A\delta x \rho \times c \times \frac{d\theta}{dt} \quad \text{---(2)}$$

Let E is the emissivity of the surface of the bar and p the perimeter of its cross section, then the heat escaped out per sec from the sides of the element between L and M will be.

$$E \times p\delta x \times \theta \quad \text{---(3)}$$

From equation (1), (2) and (3) give

$$KA\frac{d^2\theta}{dx^2}\delta x = A\delta x \rho \times c \times \frac{d\theta}{dt} + E \times p\delta x \cdot \theta$$

$$\text{or, } \frac{K}{\rho c}\left(\frac{d^2\theta}{dx^2}\right) = \frac{d\theta}{dt} + \frac{Ep}{A\rho c}\theta$$

$$\text{or, } \frac{d\theta}{dt} = \frac{K}{\rho c}\left(\frac{d^2\theta}{dx^2}\right) - \frac{Ep}{A\rho c}\theta \quad \text{---(4)}$$

This is the Fourier's differential equation for one dimensional flow of heat.